
**THEORETICAL AND EXPERIMENTAL INVESTIGATION
OF THE FUNCTION OF THE WALL FLOW DEFLECTING RING.
A GENERALIZED MATHEMATICAL MODEL FOR THE CASE
OF A LARGE NUMBER OF DEFLECTING RINGS**

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Based on the probability theory considerations a probability density distribution function has been derived for the radius on which the liquid, upon hitting the wall flow deflecting ring, or an element of packing resting on it, is deflected and proceeds descending in a trickle bed column. The obtained probability density distribution function has been used in turn in the model describing the distribution of liquid in columns equipped with the wall flow deflecting rings. The ultimate goal is a reliable theory for optimization of the size and spacing of the wall flow deflecting rings in packed bed columns.

The wall flow deflecting rings¹ (WFDR) placed in the vicinity of the walls of packed bed columns in a given spacing can substantially reduce the fraction of liquid that flows down the surface of the wall in the form of the wall flow. This in turn considerably improves conditions for interfacial transfer and makes the column performance independent of its diameter².

An earlier paper³ presented a mathematical model for the case of a single WFDR in the column and confirmed experimentally its adequacy. As the next step, the following paper⁴ presented the model for the case of a large number of WFDRs. A recurrent formula was obtained permitting, sequentially in a ring-to-ring manner, coefficients to be obtained of the solution determining the distribution of liquid in an arbitrary position in the column equipped with the WFDRs. In the model it was assumed that all liquid that hits a given WFDR drains from its inner circumference. This assumption was found correct for the case of a single WFDR (ref.³) above which there was no packing. However, in case of the WFDRs located within the bed the wall flow deflected by the WFDR does not drain from the WFDR on its inner periphery, but, instead, it drains *via* the packing elements contacting directly the WFDR (ref.⁴). This was confirmed also by visual observation and caused that

the presented model was found inadequate⁴. The resulting effect of the WFDRs is thus stronger than predicted by the model.

The aim of this paper is to present a mathematical model of the process of liquid flow distributions in the case of a large number of WFDRs taking into the account the interaction of the WFDRs with the neighbouring particles of the packing within the packed layer.

THEORY

The Mathematical Model of the Process

The fundamentals of the mathematical model to be presented are identical with those published in ref.⁴. The principal equation governing the distribution of liquid in a packed column under axial symmetry is following⁵:

$$\frac{\partial^2 f(r, Z)}{\partial r^2} + \frac{1}{r} \frac{\partial f(r, Z)}{\partial r} = \frac{\partial f(r, Z)}{\partial Z} \quad (1)$$

and is solved for the boundary conditions of Kolář and Staněk⁶:

$$-\frac{\partial f(r, Z)}{\partial r} = B[f(r, Z) - CW], \quad r = 1. \quad (2)$$

The solution then takes the following form:

$$f(r, Z) = A_0 + \sum_n A_n J_0(q_n r) \exp(-q_n^2 Z), \quad (3)$$

where

$$A_0 = C/(1 + C) \quad (4)$$

and where q_n are roots of the following characteristic equation:

$$\left(\frac{2C}{q_n} - \frac{q_n}{B}\right) J_1(q_n) + J_0(q_n) = 0. \quad (5)$$

The coefficients A_n are determined from the initial condition, i.e. by the initial liquid distribution function $\gamma(r)$ ⁷

$$A_n = \frac{2((q_n^2/B) - 2C)^2}{[((q_n^2/B) - 2C)^2 + q_n^2 + 4C] J_0^2(q_n)} \int_0^1 \gamma(r) r J_0(q_n r) dr. \quad (6)$$

The ratio dN/N expresses the probability that the liquid will reach the zone of width dr :

$$\frac{dN}{N} = d\Phi(r) = \varphi(r) dr, \quad (8)$$

where $\varphi(r)$ represents the probability density of the radius r .

Considering Eqs (7) and (8) one obtains the following expressions for the density of irrigation $f_B(r)$:

$$f_B^{(k+1)}(r) = \frac{dQ}{2r dr} = \frac{Q^{(k+1)}}{2r} \varphi(r). \quad (9)$$

Thus the initial condition for the $(k + 1)$ -th WFDR shall be as follows:

$$\begin{aligned} \gamma(r, Z) &= f^{(k)}(r, Z)_{Z=Z_0} = \\ &= A_0 + \sum_n A_n^{(k)} J_0(q_n r) \exp(-q_n^2 Z_0), \quad \text{for } Z = 0 \quad \text{and} \quad 0 \leq r < r_1 - d \quad (10) \end{aligned}$$

$$\begin{aligned} \gamma(r, Z) &= f^{(k)}(r, Z)_{Z=Z_0} + f_B^{(k+1)}(r) = \\ &= A_0 + \sum_n A_n^{(k)} J_0(q_n r) \exp(-q_n^2 Z_0) + \frac{Q^{(k+1)}}{2r} \varphi(r) \quad (11) \end{aligned}$$

$$\text{for } Z = 0 \quad \text{and} \quad r_1 - d \leq r \leq r_1,$$

$$\gamma(r, Z) = 0, \quad \text{for } Z = 0 \quad \text{and} \quad r_1 < r \leq 1. \quad (12)$$

In addition the mass balance mandates that

$$2 \int_0^1 f^{(k)}(r, Z)_{Z=Z_0} r dr = 1. \quad (13)$$

Using this balance one can determine the amount of liquid that hits the $(k + 1)$ -th WFDR as

$$\begin{aligned} Q^{(k+1)} &= 1 - 2 \int_0^{r_1} f^{(k)}(r, Z)_{Z=Z_0} r dr = \\ &= 1 - 2 \int_0^{r_1} [A_0 + \sum_n A_n^{(k)} J_0(q_n r) \exp(-q_n^2 Z_0)] r dr. \quad (14) \end{aligned}$$

Upon applying the initial conditions (10)–(12) to Eq. (6) and upon solving the integral as shown in ref.⁴ one obtains the following recurrent formula for the determination of the coefficients of the solutions (3) in the section of the packing below

the $(k + 1)$ -th WFDR:

$$\begin{aligned}
 A_n^{(k+1)} = & \frac{2((q_n^2/B) - 2C)^2}{[(q_n^2/B) - 2C]^2 + q_n^2 + 4C} J_0^2(q_n) \left\{ \frac{Cr_1}{(1+C)q_n} J_1(q_n r_1) + \right. \\
 & + \sum_{m \neq n} r_1 A_m^{(k)} \exp(-q_m^2 Z_0) \frac{1}{q_m^2 - q_n^2} [q_m J_0(q_n r_1) J_1(q_m r_1) - q_n J_0(q_m r_1) J_1(q_n r_1)] + \\
 & \left. + A_n^{(k)} \exp(-q_n^2 Z_0) \frac{r_1^2}{2} [J_0^2(q_n r_1) + J_1^2(q_n r_1)] + \right. \\
 & \left. + \frac{1}{2} \left[1 - \frac{Cr_1^2}{1+C} - 2r_1 \sum_n A_n^{(k)} \exp(-q_n^2 Z_0) \frac{J_1(q_n r_1)}{q_n} \right] \int_{r_1-d}^{r_1} \varphi(r) J_0(q_n r) dr. \quad (15)
 \end{aligned}$$

The obtained expression in Eq. (15) appears to be the most general solution for a column with a large number of WFDRs. However, the expression contains the so far undetermined distribution of the probability density of the radius, $\varphi(r)$, in the zone $r_1 - d \leq r \leq r_1$. In the particular case of initial distribution of liquid given by the disc distributor of radius equal to the inner radius of the WFDR, the Eq. (15) transforms into the expression analogous to that published in ref.⁴.

The problem now reduces to one of determining the probability density distribution $\varphi(r)$. This can be accomplished only *via* the theory of probability.

The Probabilistic Model of Irrigation in the Zone of the WFDR

Let us inspect now the vicinity of WFDR with a packing element (Raschig ring) resting on it in an arbitrary position. The situation is sketched in Fig. 2 in a plane

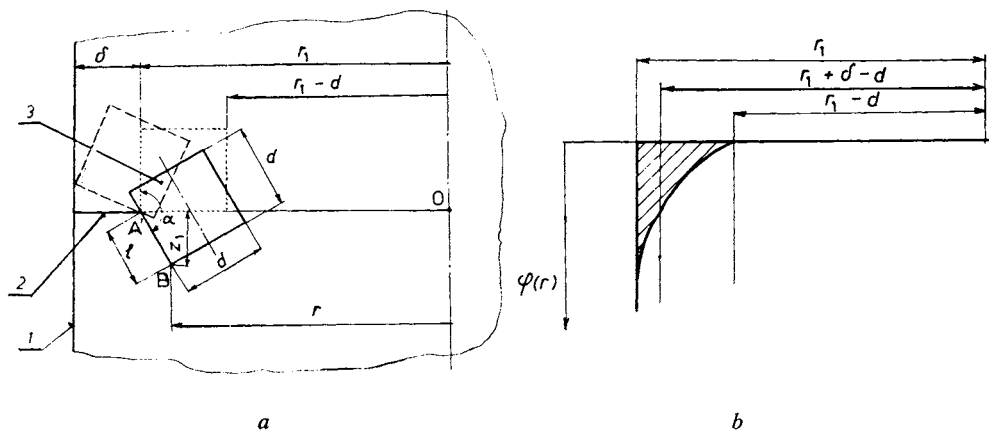


FIG. 2

Probabilistic model. *a* Scheme for the derivation; 1 column wall, 2 deflecting ring, 3 element of packing. *b* Typical course of the distribution of the probability density (r)

passing through the axis of the column. For simplicity we shall investigate the two-dimensional case assuming that the axis of the Rachig ring falls into the same plane.

Consider now polar coordinates with the center at the point A where the packing contacts the periphery of the WFDR. All possible positions are then determined by two variables l and α . The liquid flows down from the point B at a distance r from the column axis. From the standpoint of the radius r one can distinguish two zones: a) The zone $r_1 - d \leq r < r_1 + \delta - d$; i.e. in the whole interval of variables l and α the element of packing does not touch the column wall. b) The zone $r_1 + \delta - d \leq r \leq r_1$; i.e. there exist combinations of l and α for which the packing element touches the wall of the column.

It is now useful to inspect the two cases separately.

a) The zone $r_1 - d \leq r < r_1 + \delta - d$. In this zone all possible positions of the packing element, in which this element drains liquid from the region of WFDR, are determined by the following intervals of variables l and α :

$$l \in [0, d]; \quad \alpha \in [\pi/2, \pi]. \quad (16)$$

In addition, these two quantities are independent and may assume arbitrary values from the above shown intervals with equal probability (uniform distribution). Consequently, their probability distributions are as follows:

$$\varphi(l) = 1/d; \quad \varphi(\alpha) = 2/\pi. \quad (17)$$

From the independence of l and α there follows that their joint probability density distribution shall be

$$\varphi(l, \alpha) = \varphi(l) \cdot \varphi(\alpha) = 2/(d\pi). \quad (18)$$

Upon changing to the cylindrical system of coordinates, with the coordinates r and Z relating to the axis of the column, we obtain:

$$\varphi(r, Z_1) = \varphi(l, \alpha) |J|, \quad (19)$$

where the Jacobian J is determined by⁸

$$J = \begin{vmatrix} \frac{\partial l}{\partial r} & \frac{\partial l}{\partial Z_1} \\ \frac{\partial \alpha}{\partial r} & \frac{\partial \alpha}{\partial Z_1} \end{vmatrix}. \quad (20)$$

From the geometry considerations in Fig. 2 it is seen that

$$\begin{aligned} r &= r_1 - l \sin \alpha \\ Z_1 &= l \cos \alpha . \end{aligned} \quad (21)$$

Upon determining the derivatives and substituting into Eq. (20) we obtain

$$J = 1/\sqrt{[(r_1 - r)^2 + Z_1^2]} . \quad (22)$$

Then from Eqs (18), (19) and (22) there follows that the joint probability density of r and Z_1 shall be

$$\varphi(r, Z_1) = \frac{2}{d \pi \sqrt{[r_1 - r]^2 + Z_1^2}} . \quad (23)$$

The probability density of the radius $\varphi(r)$ is determined by the integral

$$\varphi(r) = \int_{Z_{1 \min}}^{Z_{1 \max}} \varphi(r, Z_1) dZ_1 . \quad (24)$$

From the intervals of the quantities l and α , Eq. (16), and from Eq. (21) there follows that in the zone $r_1 - d \leq r < r_1 + \delta - d$ we have

$$Z_{1 \min} = 0 ; \quad Z_{1 \max} = \sqrt{[d^2 - (r_1 - r)^2]} . \quad (25)$$

Thus upon solving the integral (24) we obtain

$$\varphi(r) = \frac{2}{d \pi} \ln \frac{d + \sqrt{[d^2 - (r_1 - r)^2]}}{r_1 - r} . \quad (26)$$

b) Zone $r_1 + \delta - d \leq r \leq r_1$. In this zone one can distinguish two cases: 1) the values of l and α are such that the packing element does not touch the column wall; 2) the packing element touches the wall.

In the first case l and α are independent, while in the second case they depend one from another. The probability density, $\varphi(r)$, then takes the form

$$\varphi(r) = \varphi_1(r) + \varphi_2(r) , \quad (27)$$

where the subscript 1 and 2 refer to the region of independence and dependence of l and α respectively.

The probability density $\varphi_1(r)$ is determined analogously to the previous zone by the integral (24), with the only difference that the geometry of the problem (see Fig. 2) dictates that

$$Z_{1\min} = \frac{r_1 - r}{r_1 + \delta - r} \sqrt{[d^2 - (r_1 + \delta - r)^2]};$$

$$Z_{1\max} = \sqrt{[d^2 - (r_1 - r)^2]}. \quad (28)$$

Then

$$\varphi_1(r) = \frac{2}{d\pi} \ln \frac{(r_1 + \delta - r) \{d + \sqrt{[d^2 - (r_1 - r)^2]\}}{(r_1 - r) \{d + \sqrt{[d^2 - (r_1 + \delta - r)^2]\}}}. \quad (29)$$

When the element of packing touches the wall the quantities l and α are constrained by a functional relationship. Thus $r = r(l)$ and the result takes the form of a unidimensional random variable

$$\varphi_2(r) = \varphi_2[l(r)] \left| \frac{\partial l}{\partial r} \right|. \quad (30)$$

The problem thus reduces to one of determining $\varphi_2(l)$ in the zone of contact with the wall, where this probability density is already nonuniform. To tackle this problem let us utilize the following considerations:

The joint probability density $\varphi(l, \alpha)$, when l and α are dependent, is as follows⁹

$$\varphi(l, \alpha) = \varphi(l) \varphi(\alpha/l) = \frac{1}{d} \varphi(\alpha/l), \quad (31)$$

where $\varphi(\alpha/l)$ designates conditional probability density of α .

Let us, for a given l , the limiting value of the angle, for which the packing element touches the wall, be α_1 . Its value is determined by geometry considerations (see Fig. 2) as:

$$\alpha_1 = \pi - \arcsin \frac{r_1 + \delta - r}{d}. \quad (32)$$

With respect to Eqs (24) and (31) we then get

$$\varphi_2(l) = \int_{\pi/2}^{\alpha_1} \varphi(l, \alpha) d\alpha = \frac{1}{d} \int_{\pi/2}^{\alpha_1} \varphi(\alpha/l) d\alpha. \quad (33)$$

We know⁹, however, that

$$\int_{\pi/2}^{\pi} \varphi(\alpha/l) d\alpha = 1. \quad (34)$$

Upon dividing the integral (33) into two parts by a limit α_1 , we get

$$\int_{\pi/2}^{\alpha_1} \varphi(\alpha/l) d\alpha + \int_{\alpha_1}^{\pi} \varphi(\alpha) d\alpha = 1. \quad (35)$$

Within the limits $[\alpha_1, \pi]$ of the second integral, the packing element does not touch the wall and therefore $\varphi(\alpha/l) = \varphi(\alpha) = 2/\pi$. Upon multiplying Eq. (35) by $1/d$, putting the second integral on the right hand side and considering Eqs (32) and (33) we receive

$$\varphi_2[l(r)] = \frac{1}{d} - \frac{2}{\pi d} \arcsin \frac{r_1 + \delta - r}{d}. \quad (36)$$

From the geometry considerations there follows that in case of the contact with the wall

$$l = \frac{d(r_1 - r)}{r_1 + \delta - r} \quad \text{and} \quad \left| \frac{\partial l}{\partial r} \right| = \frac{d\delta}{(r_1 + \delta - r)^2}. \quad (37)$$

Then from Eqs (37) and (36), after substituting into Eq. (30), we obtain

$$\varphi_2(r) = \left[1 - \frac{2}{\pi} \arcsin \frac{r_1 + \delta - r}{d} \right] \frac{\delta}{(r_1 + \delta - r)^2}. \quad (38)$$

In this way, in accord with Eq. (27), the probability density $\varphi(r)$ in the zone $r_1 + \delta - d \leq r \leq r_1$ shall be

$$\begin{aligned} \varphi(r) = & \frac{2}{d\pi} \ln \frac{(r_1 + \delta - r) \{d + \sqrt{[d^2 - (r_1 - r)^2]}\}}{(r_1 - r) \{d + \sqrt{[d^2 - (r_1 + \delta - r)^2]}\}} + \\ & + \left[1 - \frac{2}{\pi} \arcsin \frac{r_1 + \delta - r}{d} \right] \frac{\delta}{(r_1 + \delta - r)^2}. \end{aligned} \quad (39)$$

Typical profile of $\varphi(r)$ for the whole interval $r_1 - d \leq r \leq r_1$, determined by the dependences (26) and (39), is shown in Fig. 2b.

As already said in the derivation of the model the axis of the packing element is expected to fall into the plane passing through the column axis. In order to examine the effect of this simplification a three dimensional model has been considered, taking into account possible rotation of the packing element in the horizontal plane.

The model yielded a very cumbersome expressions while, on the other hand, the deviations did not exceed 2%.

RESULTS

In order to verify the above model the experiments from ref.⁴ were used. These experiments were carried out in a column 188.6 mm in diameter packed with Raschig rings 25 × 25 × 3 mm. Below the packing there was a collecting device consisting of 4 concentric segments delimited by radii summarized in Table I.

Three configurations were investigated with two and three WFDRs spaced 100 and 200 mm apart. Six parallel experiments were carried out for two densities of

TABLE I
Radii (mm) delimiting the collecting segments

Radius	Designation of segment			
	I	II	III	IV
Inner	0	40.2	55.3	88.9
Outer	40.2	55.3	88.9	94.3

TABLE II
Results of experiments

No. of exp.	<i>t</i> mm	No. of WFDR	<i>h</i> ₀	No. of segment	\bar{f}_i	\bar{f}_{ic}	ε %
1	20	3	100	I	1.38	1.28	8.0
1	20	3	100	II	1.26	1.30	-2.8
1	20	3	100	III	0.87	0.84	4.0
1	20	3	100	IV	0.71	0.92	-22.4
2	20	2	100	I	1.27	1.18	7.5
2	20	2	100	II	1.28	1.26	1.4
2	20	2	100	III	0.83	0.86	-3.9
2	20	2	100	IV	0.99	0.99	0.0
3	20	2	200	I	1.18	1.16	2.0
3	20	2	200	II	1.13	1.08	4.4
3	20	2	200	III	0.81	0.76	7.2
3	20	2	200	IV	1.41	1.82	-22.5

irrigation ($1.67 \cdot 10^{-3} \text{ m}^3/(\text{m}^2\text{s})$ and $3.3 \cdot 10^{-3} \text{ m}^3/(\text{m}^2\text{s})$). In each case the width of the WFDR was 20 mm.

The variance of reproducibility experiments was $S_0^2 = 0.0534$ with the number of the degrees of freedom 52.

The calculations were carried out on a digital computer. The integral in Eq. (15) was evaluated with the aid of the Simpson rule. In region $r_1 + \delta - d \leq r \leq r_1$ the function behind the integral, Eq. (29), has a singularity as the limit of $\varphi_1(r)$ for r approaching r_1 goes to infinity. In order to avoid large errors during numerical

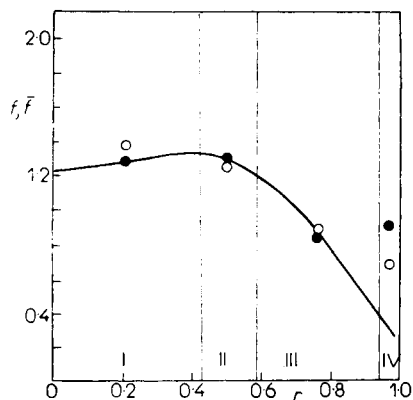


FIG. 3

A comparison of computed and measured results for three deflecting rings 100 mm apart

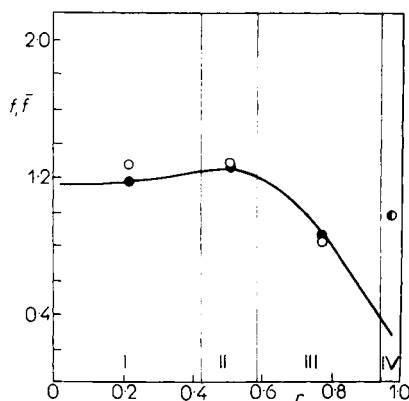


FIG. 4

A comparison of computed and measured results for two deflecting rings 100 mm apart

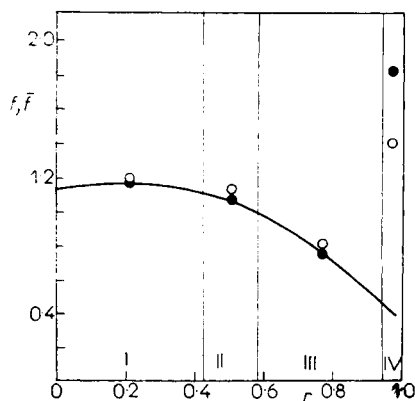


FIG. 5

A comparison of computed and measured results for two deflecting rings 200 mm apart

evaluation the following preliminary numerical test was carried out on the basis of the condition

$$\int_{r_1-d}^{r_1} \varphi(r) dr = 1. \quad (40)$$

Using parallel analytical and Simpson's numerical evaluation, the division of the interval of integration was determined, together with the approach to the limit r_1 , for which the error of evaluation of the integral did not exceed 0.2%.

The value of D in the calculations was taken³ equal 0.00235 m, $B = 7.0$ and $C = 1.365$ (ref.¹⁰). For all experimental conditions the mean density of irrigation $\bar{f}_{i,c}$ in each collecting segment was evaluated. For the outermost segment (IV) the density of irrigation was computed on the basis of the flow rate into this segment plus the wall flow.

The results of the calculations and the experiments are summarized in Table II. The same results are presented graphically in Figs 3–5 (black points indicate computed mean density of irrigation, empty points indicate the experimental values). Continuous line in these figures indicates the distribution of local density of irrigation as a function of radius.

The residual variance is determined by¹¹

$$S_A^2 = \frac{1}{P-1} \sum_{i=1}^P n_i (\bar{f}_i - \bar{f}_{i,c})^2 \quad (41)$$

and for given results $S_A^2 = 0.090$ with 11 degrees of freedom.

From the Fischer criterion¹¹

$$F = S_A^2/S_0^2 = 1.69 < F_{1-\alpha/2} = 2.0. \quad (42)$$

Then there follows that the model is adequate at the significance level $\alpha = 0.1$.

LIST OF SYMBOLS

A_0, A_n	coefficients in Eq. (3)
B, C	dimensionless coefficients of boundary condition (2)
D	coefficient of radial spread of liquid, m
d_p	diameter of element of packing (Rashig ring), m
$d = d_p/R$	dimensionless diameter of Raschig ring
F	Fischer criterion
$f = L/L_0$	dimensionless density of irrigation
\bar{f}	mean dimensionless density of irrigation
h	height of packed section measured from lower WFDR, m
h_0	WFDR spacing, m

J	Jacobian, Eq. (20)
J_0, J_1	Bessel function of the first kind, zero and first order
k	index number of WFDR from the uppermost WFDR
L, L_0	local and mean density of irrigation, $m^3/(m^2 s)$
$l = l'/R$	dimensionless length of the packing element outside the periphery of the WFDR
l'	length of the packing element outside the periphery of the WFDR, m
m	summation index
N	number of packing elements draining liquid from WFDR
n	summation index; number of parallel experiments
P	number of configurations per number of segments
Q	flow rate of liquid that hits WFDR
q_n	roots of Eq. (5)
R	column radius, m
r'	radius, m
$r = r'/R$	dimensionless radius
r_1	dimensionless radius of inner periphery of WFDR
S^2	estimate of variance
t	width of WFDR, m
W	dimensionless wall flow
$Z = Dh/R^2$	dimensionless coordinate of height
$Z_1 = h/R$	dimensionless coordinate in Descartes system of coordinates
$Z_0 = Dh_0/R^2$	dimensionless spacing of the WFDRs
α	angle of inclination of axis of packing element in the vertical plane (Fig. 2)
γ	initial dimensionless density of irrigation distribution
$\delta = t/R$	dimensionless width of the WFDR
ϵ	relative deviation of the experimental and calculated mean density of irrigation for the i -th segment, %
φ	distribution of probability density
$d\Phi$	element of probability

Superscripts

k for the k -th WFDR

Subscripts

A residual variance
 c calculated value
 i for the i -th segment
 0 reproducibility variance

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